

Quantum Gravity and the Problem of Measurement

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We discuss the proposal that the quantum correlations contained in the pure state-vector evolving according to Schrödinger equation can be eliminated by the action of multiply connected wormholes during measurement. A procedure is devised to obtain a proper master equation which governs the changes of the reduced density matrix of matter fields interacting with doubly connected wormholes. It is shown that this master equation predicts an appropriate damping of the off-diagonal correlations contained in the state vector.

1. THE PROPOSAL

There have been some proposals to provide the necessary decay of quantum coherence during measurement with a physical explanation (Zurek, 1991). The most popular among these proposals is the so-called decoherence program advocated mainly by Zeh (1970), Gell-Mann and Hartle (1990), and Unruh and Zurek (1989). It is not quite clear, however, that these proposals could provide a consistent mechanism to generate the causal nonlocality and arrow of time which are also induced by quantum measurement (Bell, 1975; Omnès, 1994). Another approach to the quantum measurement problem has been advocated by Penrose (1987). It ascribes the cause leading to the wave packet collapse to some still unspecified form of gravitational interaction. In this paper we shall consider a nonunitary evolution of microscopic systems arising from a violation of causal locality which can only be induced by *non-simply* connected wormholes (González-Díaz, 1992).

Wormholes are microscopic connections between two otherwise disconnected flat regions of spacetime and represent a topology change in that they induce an initial state, which is just flat space, to evolve into a final state which is flat space plus a given number of baby universes (Hawking, 1990).

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We distinguish two possible inner topologies which may appear in wormholes. If the inner topology is simply connected, the corresponding quantum state is a pure state described by a wave function (Hawking, 1990). However, if the inner topology of the wormholes is not simply connected, the quantum state becomes mixed and should be described by a density matrix (González-Díaz, 1991).

The interaction formalism worked out by using the technique of the Green function filtered by the wormhole quantum state (González-Díaz 1991, 1993a, b) gives rise to a bilocal interaction contribution $P_i I_{bl}(x_1, x_2; y_1, y_2)$, where the bilocal factor I_{bl} is independent of the wormhole state. The prefactor P_i depends, however, on that state. If we start with a wormhole wave function, $P_i \equiv P_\Psi$ is just an unimportant numerical coefficient of order unity, but if we choose a density matrix as the wormhole state, then $P_i \equiv P_{\rho_b}(n_j)$ will depend on the wormhole energy spectrum (González-Díaz 1993a, b, 1994a, b, 1995). The quantum state of wormholes is generally given by a path integral

$$F_w = \int_C d[g_{\mu\nu}] d[\Phi_0] e^{-I[g_{\mu\nu}, \Phi]} \quad (1.1)$$

in which I is the Euclidean action and C represents the class of asymptotically flat Euclidean four-geometries and asymptotically vanishing matter-field configurations which match either the prescribed data on a three-surface dividing the four-manifold in the case of a pure state given by a wave function Ψ (Hawking, 1988), or the data on its three-surface and the reverse orientation of the corresponding set of data on its copy three-surface in the case of a mixed state given by a density matrix ρ_b (González-Díaz, 1991). If $F_w = \Psi$, one can apply the Gell-Mann–Low formula and obtain an effective interaction Hamiltonian given by the Hawking–Coleman expression (Hawking, 1988; Coleman, 1988a, b), $\sum_k H_k^L(\Phi) (a_k^\dagger + a_k)$, where the discrete index k collectively labels the types of different baby universes, the H_k^L are matter-field interaction Hamiltonian terms, and the a_k are Fock operators for the baby universes. This leads (Hawking, 1990) to no loss of quantum coherence and implies (González-Díaz, 1994a, b) that the quantum state of a simply connected wormhole contains an equal contribution from complex and their complex conjugate metrics, so that causal locality (i.e., $[H^I(x), H^I(y)] = 0$) holds (Hawking, 1990) and both Ψ and the quantum state of the matter field should then be time-symmetric. On the contrary, if $F_w = \rho_b$, one cannot apply the Gell-Mann–Low formula. Using then a combinatorial procedure, we obtain (González-Díaz, 1992) $\sum_k H_k^L(\Phi) A_k(a_k^\dagger, a_k)$, where A_k generally contains higher, nonlinear powers of the baby-universe Fock operators. Restricting ourselves to the simplest case in which the inner wormhole topology is doubly connected (González-Díaz, 1992), we have $A_k \simeq a_k^{+2} + a_k^2$ and

$$\int d^1x_0[H^I(x), H^I(x^I)] = D_0 \left(c^\dagger c + \frac{1}{2} \right) \sinh(2k_0) \quad (1.2)$$

where D_0 is a constant of order unity, with the c 's being Fock operators for the matter field, and $k_0 = (2k/(R_0^2 - k))^{1/2}$, in which $(2k)^{1/2}$ denotes the proper separation distance, measured on the wormhole inner three-manifold, between the two correlated points at which two baby universes are created or annihilated, two at a time, and $(R_0^2 - k)^{1/2}$ gives the length scale of the baby universes. Then the demand of Lorentz invariance implies (González-Díaz, 1992, 1994a, b) that (1.1) leads to the loss of quantum coherence so as to a breakdown of time symmetry and causal locality, at least for CP -invariant matter with positive energy. Thus, non-simply connected wormhole fluctuations are able to induce all the effects which are required for quantum measurements. We summarize then our proposal as follows. The unitary evolution of the state vector governed by the Schrödinger equation should be associated with a physical system interacting with wormholes which are all in a pure state, while the nonunitary quantum-measurement evolution should correspond to a physical situation in which the system interacts with at least a nonzero proportion of wormholes in the mixed state.

Our proposal needs to be supplemented, however, in two important respects. First of all, it is not still quite clear whether the loss of quantum coherence induced by the breakdown of casual locality implied by (1.2) is of the kind required by quantum measurement to damp the off-diagonal correlations in the state vector in a sufficiently short decoherence time. This question will be addressed in the next sections. Second, one would need to provide the scenario with a reasonable mechanism by which one could answer the question, "why should one suddenly replace simply connected wormholes by multiply connected wormholes when some quantum measurement is being carried out on a system?" We shall devote the rest of the present section to briefly comment on this question. Wormholes were initially claimed (Coleman, 1988a, b) to fix the observed values of all physical and cosmological constants. Actually, this can only be the case when multiply connected wormholes are considered; otherwise, the topological fluctuations induce unphysical values for these constants, such as either zero or infinite values for the physical constants or, more importantly, a more probable large negative value for the cosmological constant. It is only for statistical wormholes that the whole set of observable effects induced by them on ordinary matter at low energies can be thought to produce some anthropic consequences, along with observability of the matter states and their couplings (González-Díaz, 1993a, b). Indeed, of the set of all possible values of the physical and cosmological constants, such wormholes should take on those values which will satisfy the requirement that there exist sites in the universe where carbon-based life

can evolve, thus rendering any form of anthropic principle (Barrow and Tipler, 1986) just an effect induced by statistical wormholes on matter. On the other hand, it has been shown (González-Díaz, 1994a, b) that it is just the purely statistical content of single mixed wormhole states which can give rise to the emergence of a consistently defined cosmological time concept. There is no similar time concept arising from wormholes when they are in a pure state. The above question then becomes meaningless, since asking about anything prior to the emergence of time does not make any sense. Moreover, the time asymmetry which is also induced by single wormhole statistical states can be regarded as the common physical origin for all existing time arrows, including the so-called psychological arrow by which observers are able to remember just those records in their memory produced by measurements already performed. It appears then that there is some strong link—if not direct correspondence—between the appearance of non-simply connected wormholes and both the existence of observers able to perceive the flow of time and record the results of measurements, and the simultaneous emergence of a causally connected *classical* reality endowed with a set of observables.

2. THE MASTER EQUATION

We shall restrict ourselves to the simplest case of interaction between a massless, conformally invariant scalar matter field $\Phi(x, t)$ and single, doubly connected wormholes (González-Díaz, 1992), and use the formalism of the density matrix in the interaction representation and ordinary time-dependent perturbation theory in first-order approximation. We start with an interaction Hamiltonian $H^I = \sum_i H_i^I(\Phi) A_i$ and assume a full density matrix $\rho = \rho_\Phi \otimes \rho_b = \rho_\Phi \sum_{i,j} |i\rangle\langle j|$, where ρ_Φ and ρ_b are density matrices for the scalar matter field and the baby universes, respectively. From the equation of motion for ρ , which we iterate for small increment of time, we obtain with the same approximation as used in ordinary time-dependent perturbation calculations,

$$\dot{\rho}(k, t) = Tr_b \sum_{i,j} [H_i^I A_i, [H_j^I A_j, \rho]] \quad (2.1)$$

in which Tr_b means tracing over the baby universe operators.

Also, we assume an orthonormalization relation

$$\langle j|i\rangle = \delta(2k - (x - x')^2) \delta(p^2 - (R_0^2 - k)^{-1}) \quad (2.2)$$

and commutation relations for hermitian operators A_i (González-Díaz, 1992)

$$\int d^4x_0 [A(x), A(x')] = E_0 \left(a^\dagger a + \frac{1}{2} \right) \sinh \left(\frac{8k}{R_0^2 - k} \right)^{1/2} \quad (2.3)$$

where $x_0 = 1/2(x - x')$ and E_0 is a constant c-number of order unity. The baby universe commutator (2.3) will consistently vanish at the limit $k \rightarrow 0$.

Using then the usual Fock expansion for quantum-field operators, and introducing the substitution (González-Díaz, 1992) $\Sigma_{i,j} \rightarrow \int d^4x_0$, we obtain, after integrating over $x_0 = 1/2(x - x')$ and momentum p , with the customary measure

$$d\tilde{P} = \frac{dp}{(2\pi)^3} \delta((p)^2 - (R_0^2 - k)^{-1}\theta)(p_0)$$

the master equation for the reduced density matrix ρ_Φ of the scalar matter field Φ :

$$\begin{aligned} \dot{\rho}_\Phi(k_0, t) = & A(k_0, N)O_4(k_0, c)\rho_\Phi + B(k_0, N)\rho_\Phi O_4(k_0, c) \\ & - 2[C(k_0, N)c^2\rho_\Phi c^{\dagger 2} + D(k_0, N)O_4^{\dagger}(c) + F(k_0, N)c^{\dagger 2}\rho_\Phi c^2] \end{aligned} \quad (2.4)$$

where $N = 0, 2, 4, \dots$ denotes the initial number of baby universes, $k_0 = (2k/(R_0^2 - k))^{1/2}$, $Q(k_0, N) = E_0 c^{-2k_0} (2N + 1) \sinh(2k_0)$,

$$O_4(k_0, c) = e^{4k_0} c^2 c^{\dagger 2} + 4 \left[(c^\dagger c)^2 + c^\dagger c + \frac{1}{4} \right] e^{2k_0} + c^{\dagger 2} c^2 \quad (2.5)$$

$$O_4^{\dagger}(c) = 2 \left(2c^\dagger c \rho_\Phi c^\dagger c + c^\dagger c \rho_\Phi + \rho_\Phi c^\dagger c + \frac{1}{2} \rho_\Phi \right) \quad (2.6)$$

$$\begin{aligned} A(k_0, N) = & (N + 1)(N + 2)e^{-4k_0} + N(N + 1) + \frac{1}{4} e^{-2k_0} \\ & + 2Q(k_0, N) \end{aligned} \quad (2.7)$$

$$B(k_0, N) = A(k_0, N) - 2Q(k_0, N) \quad (2.8)$$

$$\begin{aligned} C(k_0, N) = & (N + 1)(N + 2) + \left[N(N + 1) + \frac{1}{4} \right] \cosh(2k_0) \\ & + Q(k_0, N) \end{aligned} \quad (2.9)$$

$$D(k_0, N) = 2(N + 1)^2 \cosh(2k_0) + \frac{1}{4} + Q(k_0, N) \quad (2.10)$$

and

$$\begin{aligned} F(k_0, N) = & \left[(N + 1)(N + 2) + \frac{1}{4} \right] \cosh(2k_0) + N(N + 1) \\ & + Q(k_0, N) \end{aligned} \quad (2.11)$$

In order to eliminate changes in the reduced density matrix $\rho_\Phi(k_0, t)$ which do not originate from quantum nonlocality, we first take the limit $k \rightarrow 0$ in (2.4) and subtract then the resulting expression (which is associated with virtual processes by which scalar field quanta are created and annihilated in such a way as to contribute to the full master equation terms proportional to the Fock operator products $c^2 c^{\dagger 2}$, $c^{\dagger 2} c^2$, $c^2 \rho_\Phi c^{\dagger 2}$ and $c^{\dagger 2} \rho_\Phi c^2$) from (2.4) to finally obtain a reduced density matrix for matrix elements in the Fock space of matter-field number states, in diagonal representation $P_n(k, t)$, such that $P_n(0, t) = 0$ results from a minimal condition for the vanishing of the nonlocal effects in $P_n(k, t)$ at $k \rightarrow 0$, where

$$\bar{P}_n(k, t) = -P(N, k_0) \left(n + \frac{1}{2} \right)^2 \bar{P}_n(k, t) \quad (2.12)$$

with $P(N, k_0) = 8(N + \frac{1}{2}) \sinh(2k_0)$. Equation (2.12) consistently reduces to zero only when the parameter $k \rightarrow 0$. Note that even for the vacuum states $N = n = 0$, $P_n(k, t)$ is not time-invariant, but gives rise to a residual zero-point loss of quantum coherence.

3. THE TRANSITION FROM QUANTUM TO CLASSICAL

The density matrix $\bar{\rho}_\Phi$ in scalar particle number representation $\bar{P}(k, t)$ corresponds actually to a quantum state in position representation. The wormhole parameter $(2k)^{1/2}$ is nothing but the particular value of the spacelike separation $(x - x')$ which coincides with the proper separation distance, measured on a cross section of the inner wormhole manifold, between the two correlated points at which baby universes are created or annihilated in pairs. Hence, the evolution of the density matrix $P_n(x, x', t)$ will satisfy an accordingly generalized master equation, i.e.,

$$\bar{P}_n(x, x', t) = -P(N, x, x') \left(n + \frac{1}{2} \right)^2 \bar{P}_n(x, x', t) \quad (3.1)$$

with

$$P(N, x, x') = 8 \left(N + \frac{1}{2} \right) \sinh \left[\frac{4(x - x')^2}{r_0^2} \right]^{1/2} \quad (3.2)$$

where $r_0 = (R_0^2 - k)^{1/2}$ is the radius of the throats in the wormhole.

A coherent superposition of two Gaussians separated by a distance equal to $\Delta x = x - x'$ will now be considered (Zurek, 1991). At macroscopic distances, Δx will be much larger than the Gaussian width, so that the density matrix describing the state vector will have four peaks, two at $x = x'$ (i.e.,

the diagonal elements, which should survive the wave packet collapse) and two at $x \neq x'$ (i.e., the off-diagonal elements responsible for quantum correlations, which should disappear during the measurement process, giving rise to position as an exactly preferred basis). Clearly, as required by quantum measurement, equation (3.1) will not produce any effects on the diagonal peaks, but will induce a substantial damping of the off-diagonal peaks. The parameter N appearing in (3.1) is proportional to the initial population of baby universes. Hence, in semiclassical approximation, (3.2) can be written

$$P(N, x, x') \equiv P(S_\omega, x, x') = 8e^{-S_\omega} \sinh \left[\frac{4(x-x')^2}{r_0^2} \right]^{1/2} \quad (3.3)$$

where S_ω is the Euclidean action of the wormhole. For non-simply connected wormholes, the path integral which describes the effects of wormholes on ordinary matter is given in terms of a Planckian probability (González-Díaz, 1993a, b, 1995) $\pi(\alpha)$ for the Coleman α parameters (Hawking, 1990), and hence the semiclassical nucleation rate for baby universes,

$$e^{-S_\omega} = \alpha^2 \{2 \ln[1 + \pi(\alpha)^{-1}]\}^{-1} \quad (3.4)$$

would appear to play the role of an equilibrium temperature T_b for our wormhole–scalar particle system.

On the other hand, the factor r_0^{-1} in the argument of the hyperbolic sine in (3.3) must be proportional to the mass m acquired by the scalar particles while interacting with the wormholes, and the whole factor $P(T_b, x, x')$ would be associated with the extreme case where only wormholes which are non-simply connected are involved in the interaction. Since in quantum gravity there should also exist a given, generally nonzero contribution from simply connected wormholes, in order to account for this contribution one should modify the whole expression (3.1) by introducing an overall factor $0 \leq \gamma_b \leq 1$, interpretable as a rate of coherence loss induced by interaction with wormholes, so that the full master equation would become

$$\bar{P}_n(x, x', t) \sim -\gamma_b T_b \sinh[(x-x')m] \left(n + \frac{1}{2} \right)^2 \bar{P}_n(x, x', t) \quad (3.5)$$

Our master equation can now be compared with the term responsible for Brownian fluctuations (Zurek, 1991) of the master equation in current decoherence programs

$$\dot{\rho}_{CL}^{(f)} = -2M\gamma T(x-x')^2 \rho_{CL}^{(f)} \quad (3.6)$$

where the relaxation rate is $\gamma = \eta/\langle 2M \rangle$, with η the interaction viscosity, T the field temperature, and M the mass. It is seen that although (3.5) and (3.6)

depend on similar parameters in the same qualitative way, (3.5) shows quite stronger dependences on $(x - x')$ and mass for large values of these parameters. Note, furthermore, that (3.5) depends on the square of particle number. In any case, these two expressions damp the off-diagonal correlations in the qualitative way required by quantum measurement.

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